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463A (Geometry) [1915, 161; 1919, 414]. Proposed by B. J. BROWN, Kansas City.

If μ and ν are the parameters of the two confocal conicoids through any point on the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

show that $\mu + \nu + a^2 + c^2 = 0$, along a central circular section.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

The parameters being μ and ν , the conicoids confocal with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{1}$$

are

$$\frac{x^2}{a^2 + \mu} + \frac{y^2}{b^2 + \mu} + \frac{z^2}{c^2 + \mu} = 1 \tag{2}$$

and

$$\frac{x^2}{a^2 + \nu} + \frac{y^2}{b^2 + \nu} + \frac{z^2}{c^2 + \nu} = 1. \tag{3}$$

(1) - (2) gives

$$\frac{x^2}{a^2(a^2 + \mu)} + \frac{y^2}{b^2(b^2 + \mu)} + \frac{z^2}{c^2(c^2 + \mu)} = 0 \tag{4}$$

and (1) - (3),

$$\frac{x^2}{a^2(a^2 + \nu)} + \frac{y^2}{b^2(b^2 + \nu)} + \frac{z^2}{c^2(c^2 + \nu)} = 0. \tag{5}$$

The equation of the central circular sections of (1) is

$$c^2(a^2 - b^2)x^2 - a^2(b^2 - c^2)z^2 = 0, \tag{6}$$

$a^2 > b^2 > c^2 > 0$. Since (4), (5), and (6) must be consistent for values of x, y, z , not all zero,

$$\begin{vmatrix} \frac{1}{a^2(a^2 + \mu)}, & \frac{1}{b^2(b^2 + \mu)}, & \frac{1}{c^2(c^2 + \mu)} \\ \frac{1}{a^2(a^2 + \nu)}, & \frac{1}{b^2(b^2 + \nu)}, & \frac{1}{c^2(c^2 + \nu)} \\ c^2(a^2 - b^2), & 0, & -a^2(b^2 - c^2) \end{vmatrix} = 0.$$

After removing the factor $\mu - \nu \neq 0$, and certain other factors which are not zero by the conditions above, this determinant reduces to

$$a^2 + c^2 + \mu + \nu = 0.$$

499 (Geometry) [1916, 341; 1919, 414]. Proposed by NATHAN ALTSHILLER, University of Oklahoma.

Find the surfaces all the plane sections of which are circles.

SOLUTION BY W. D. CAIRNS, Oberlin College.

With any position of three mutually perpendicular axes, for a fixed value of x the equations of the surfaces must be of the form $l_1(y^2 + z^2, y, z) = 0$, where l_1 designates a linear form, the coefficient of $(y^2 + z^2)$ not vanishing; similarly, for $y = \text{const.}$ and for $z = \text{const.}$ they must take the forms $l_2(x^2 + z^2, x, z) = 0$ and $l_3(x^2 + y^2, x, y) = 0$, respectively. It may be shown by theorems relating to the equivalence of polynomials that this is possible only if the equations of the surfaces are of the linear form:

$$l(x^2 + y^2 + z^2, x, y, z) = 0,$$

in which the coefficients are constants; that this is sufficient is a well known fact. Hence, the class of surfaces is composed of all spheres.

Also discussed by WILLIAM HOOVER.